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MATHEMATICAL MODELLING OF THORACIC RESPONSE TO BLUNT IMPACT

Jeffrey H. Marcus, Rolf. H. Eppinger
National Highway Traffic Safety Administration

Guy S. Nusholtz
University of Michigan Transportation Research Institute

Introduction

A transfer function is a mathematical description of one signal's relationship to another. Thus, if it is known that a given turn of a car's steering wheel results in a given turning circle of the car, a simple transfer function is known. The input signal (amount the steering wheel is turned) and the transfer function predicts the output signal (the car's turning circle). Transfer functions have many uses, for not only do they serve to model and predict a system's response to various inputs, but insight about why the system responds as it does may be gained by studying the transfer function.

At the 25th Stapp Conference, Eppinger and Chan (1) presented a method of generating a transfer function between various body parts. Using several lateral cadaver impact tests, the response of an accelerometer mounted on the fourth right rib was predicted based on an accelerometer on the fourth left rib. The Biomechanic's Group of the National Highway Traffic Safety Administration (NHTSA) currently maintains a data base of signals recorded from cadaver and dummy impact tests. Software has been implemented to quickly and easily generate a transfer function between any two signals in the data base. Additional uses of the transfer function are discussed here. All tests discussed in this paper are also lateral impacts, though this is not a restriction imposed by the transfer function technique.

Theoretical Background

With the advent of large and inexpensive digital computers, extensive theory and practice related to processing information in the discrete digital domain has resulted. There shall be no attempt here to provide a complete or rigorous development of the field of digital signal processing, nor will a full and complete explanation of the methods used to generate a transfer function be offered. The interested reader is referred to Eppinger and Chan's paper (1), and standard signal processing texts such as Robiner and Gold (2).

When a system is characterized as a discrete-time system, it has the ability to convert one sequence (called the input) to another sequence (called the output). The output is related to the input by the following relationship:

$$y(n) = \emptyset[x(n)] \tag{1}$$

where:

$y(n)$ = the output

$x(n)$ = the input

$\emptyset[\ ]$ = the transfer function

It can be shown (2) that if a system is:

a. linear (i.e., when $x_1(n)$ produces $y_1(n)$ and $x_2(n)$ produces $y_2(n)$ then $ax_1(n) + bx_2(n)$ produces $ay_1(n) + by_2(n)$)

and

b. time invariant (i.e., if $x(n)$ produces $y(n)$, then $x(n-n_0)$ produces $y(n-n_0)$

then a convolutional relation exists between an input sequence, $x(n)$; a system's "impulsive response", $h(m)$; and the output response, $y(n)$, of the form

$$y(n) = \sum_{m=-\infty}^{+\infty} h(m)\ x(n-m) \tag{2}$$

It can be seen in equation (2) that the impulsive response, $h(m)$, has an infinite number of terms. A finite length approximation, $h'(m)$ exists such that

$$y'(n) = \sum_{m=1}^{M} h'(m)\ x(n-m) \tag{3}$$

= approximation of the true value of $y(n)$

Given a known input $x(n)$, and a known output $y(n)$, $h'(m)$ may be computed by minimizing the residual squared error, $e(n)$.

$$e(n) = (y(n) - y'(n))^2 \tag{4}$$

Specifically, the following matrix equation is solved

$$\begin{bmatrix} A(1,1) & A(1,2) & \cdots & A(1,M) \\ \vdots & & & \\ A(M,1) & & \cdots & \end{bmatrix} \begin{bmatrix} h'(1) \\ h'(2) \\ \vdots \\ h'(M) \end{bmatrix} = \begin{bmatrix} B(1) \\ B(2) \\ \vdots \\ B(M) \end{bmatrix} \tag{5}$$

where

$$A(i,j) = \sum_{n=M}^{N} x(n-i+1)\ x(n-j+1)$$

$$B(i) = \sum_{n=M}^{N} y(n)\ x(n-i+1)$$

M = number of coefficients in h'

N = number of discrete digitized data values

After equation (5) is solved for h', equation (3) may be used. Thus h' is the transfer function, and $y'(n)$ is the predicted output. In particular, h' may be used with a sequence $x_2(n)$ which was not used to compute h'. The resulting $y_2'(n)$ is then a predicted output resulting from the given output $x_2(n)$.

<u>Generating and Using Transfer Functions</u>

Two signals from the Biomechanic's Group's data base are shown in Figure 1. The signals are from an 8 m/s lateral pendulum impact to the thorax of a cadaver. This test was conducted at the University of Michigan Transportation Research Institute, formerly HSRI. The pendulum impactor was covered with a 4 inch thick pad of Ensolite AL. Figure 1 shows the lateral acceleration of the fourth left rib (LUR) and the force (in lbs.) acting on the pendulum impactor. The larger magnitude response is pendulum force. Note that pendulum acceleration, not pendulum force, is actually stored in the data base. Pendulum force, computed by multiplying the pendulum mass times the acceleration signal is plotted for easier plot scaling and viewing.

The LUR acceleration was designated as the input, and pendulum acceleration as the output. Using these signals a transfer function was generated. The resulting transfer function was then used in equation (3) with the LUR acceleration. The actual pendulum acceleration, and the synthetic predicted pendulum acceleration have been plotted on the same graph in Figure 2. The agreement between the two signals is quite good.

The transfer function generated contained 55 coefficients, i.e., M in equation (5) is 55. Figure 3 is a plot of the coefficients. All data used was filtered at 100 Hz, and sampled at 1600 Hz. Recalling that h'(m) is the impulsive response of a system, and that Figure 3 is a plot of h'(m), it can be seen that the transfer function behaves as intuition would predict, i.e., a damped, oscillatory response.

While the transfer function in Figure 3 contains 55 coefficients, the program which computes h'(m) can compute up to 90 coefficients. Generally, the larger the number of coefficients the more accurate y'(n) will be. The restriction of 90 coefficients is based on the central memory restrictions of the PDP-11/40 on which the program runs. The PDP-11/40, a 16 bit machine, requires programs to run in a 32K or smaller memory partition.

It would be of only academic interest to generate a transfer function and then test it with the same input signal as was used to generate the transfer function. The true utility of the transfer function would be to generate a signal not recorded in a test. For example, the Biomechanic's data base contains tests involving lateral impacts with a rigid wall. Originally, load cells were not placed behind the wall, and as a result the force input to the cadaver's thorax is not known. If a transfer function could be generated based on a test where the load into the body was known, and this transfer function validated, then it could be used to generate load information for tests where the load signal was not known.

To illustrate and validate this approach the following test of the transfer function was made. Figure 4 shows the LUR response from a lateral, cadaver sled test run at the University of Heidelberg. The cadaver impacted a rigid wall backed by load cells. The signal in Figure 4 was used as the input (i.e., $x(n)$) in Equation 3. The $h'(m)$ used was the transfer function in Figure 3, generated based on a padded pendulum impact. Figure 5 shows the computed $y'(n)$, i.e., predicted pendulum acceleration which would give the LUR acceleration in Figure 4. If the signal in Figure 5 is multiplied by the appropriate mass of the pendulum, the result is the expected force between the cadaver and the wall. Figure 5 has a signature similar to the load cell signature from Heidelberg.

Another use of the transfer function would be to determine the loading characteristics needed to produce a certain body response. Suppose that the rib is to have a 17 millisecond half-sine acceleration with a peak magnitude of 100 g's. This corresponds to a 20 mile/hour change in velocity. Such a signal was created and used as input ($x(n)$) to the transfer function from Figure 3. The impactor acceleration (assuming that impactor mass does not change between tests) which would cause the 17 ms half-sine rib response was predicted by the transfer function, and is shown in Figure 6.

Thus, if there was some optimal rib response which minimized injury, it would be possible to predict the necessary loading history to the body. Transfer functions have potential as injury countermeasure design tools.

Conclusions

This has been an early look at current work with transfer functions. Such transfer functions have been generated based on one set of test conditions, and used to predict the response to a different set of test conditions. The implication is that the systems involved are time-invariant. It is also noteworthy that a linear transfer function is used, implying that the response of the system is close to linear.

There do seem to be limits to this analysis. For example, a transfer function based on a tap test (a very low energy test) does not accurately predict the response to a high energy impact. However, these methods do seem to have much potential. Work will continue pursuing further uses of transfer functions, and examining their limitations.

References

(1) Eppinger, R.H., and Chan, H.S., "Thoracic Injury Prediction via Digital Convolution Theory," 25th Stapp Car Crash Conference, 1981, SAE.

(2) Rabiner, L., and Gold, B., Theory and Application of Digital Signal Processing, Prentiss Hall, Englewood Cliffs, New Jersey, 1975.

Figure 1

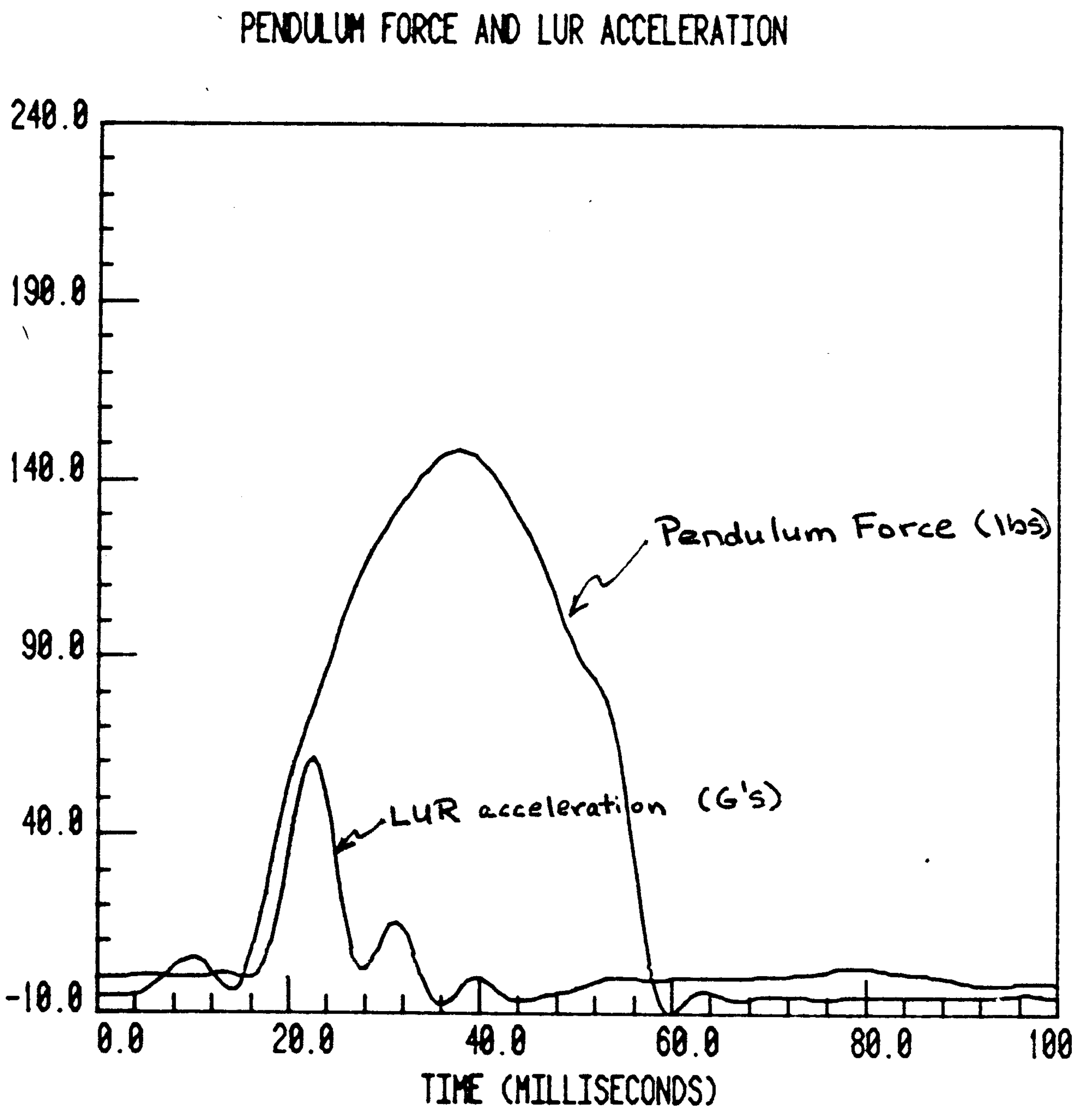

PENDULUM FORCE AND LUR ACCELERATION
240.0
190.0
140.0
90.0
40.0
-10.0
0.0
20.0
40.0
60.0
80.0
100
TIME (MILLISECONDS)
Pendulum Force (lbs)
LUR acceleration (G's)

Figure 2

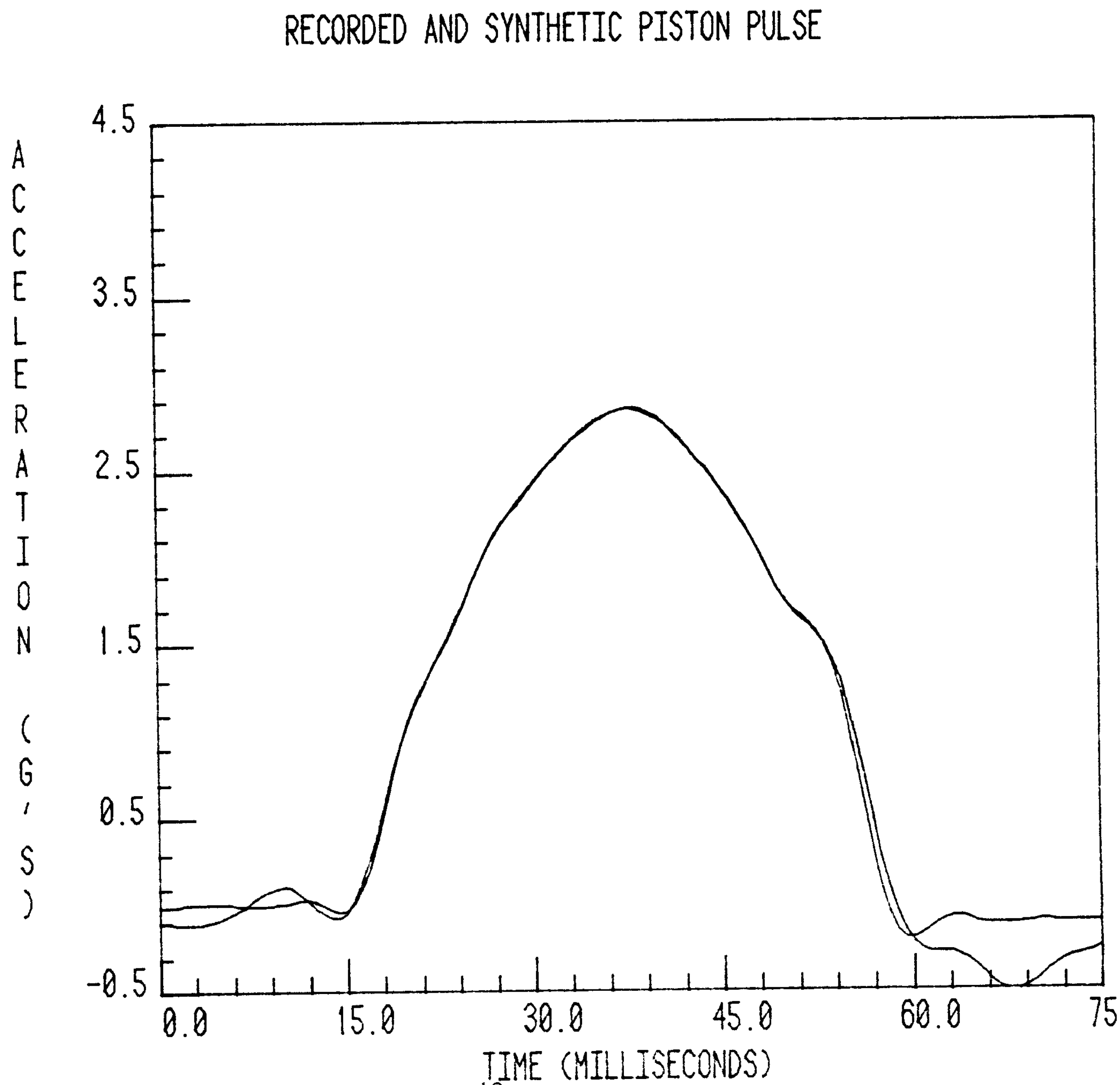

RECORDED AND SYNTHETIC PISTON PULSE
4.5
3.5
2.5
1.5
0.5
-0.5
ACCELERATION (G'S)
0.0
15.0
30.0
45.0
60.0
75
TIME (MILLISECONDS)

Figure 3

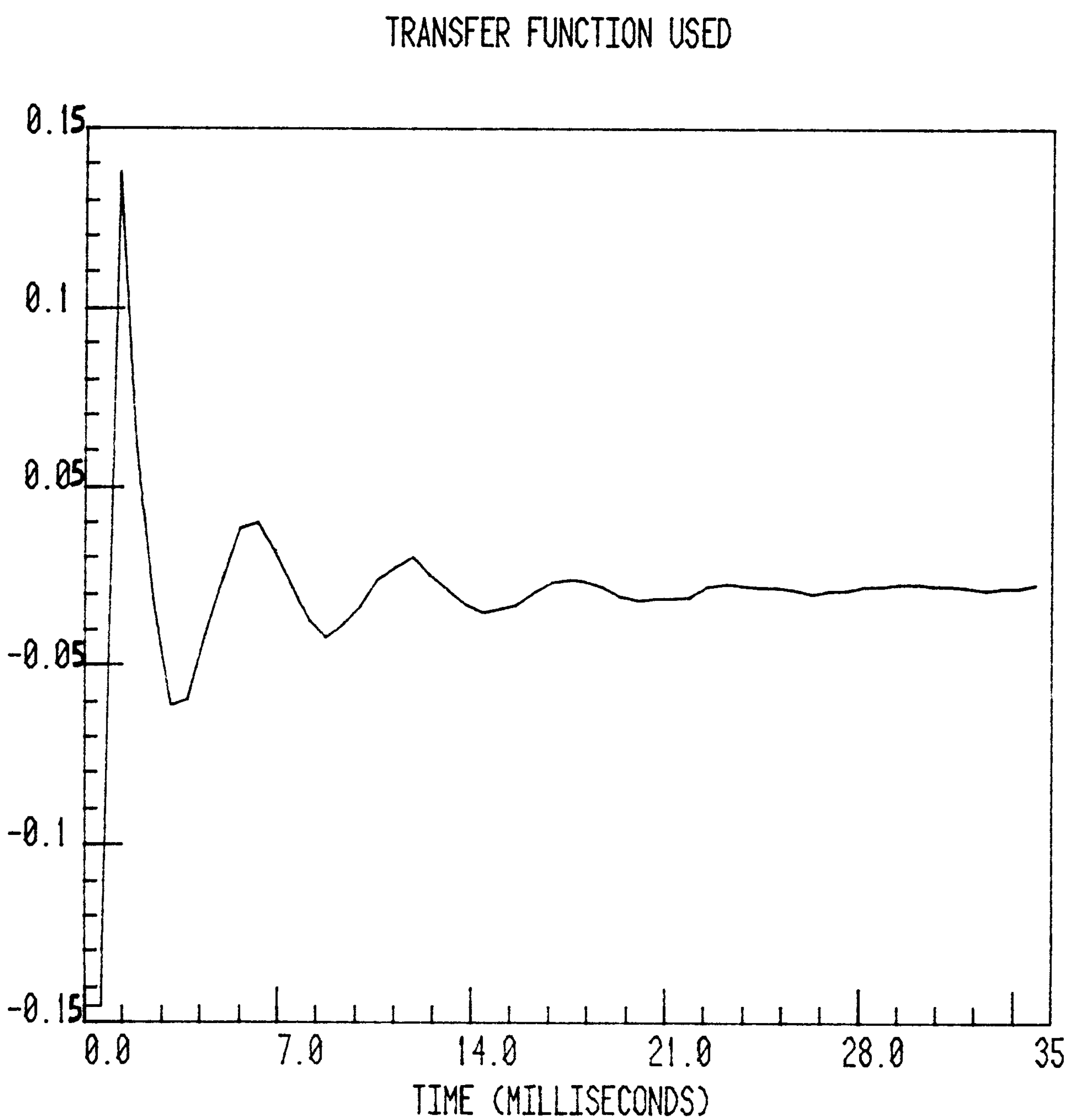

TRANSFER FUNCTION USED
0.15
0.1
0.05
-0.05
-0.1
-0.15
0.0
7.0
14.0
21.0
28.0
35
TIME (MILLISECONDS)

Figure 4

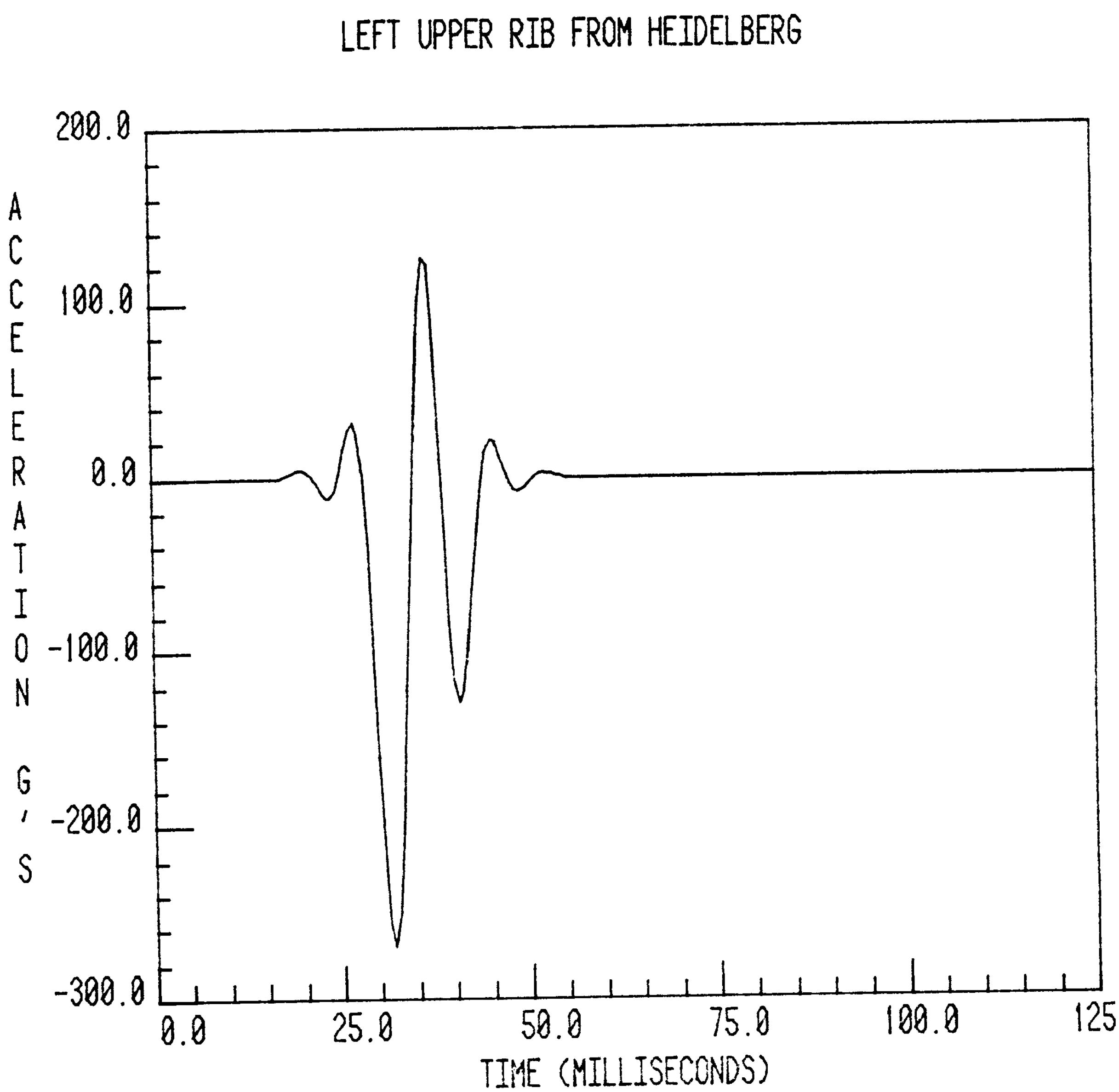
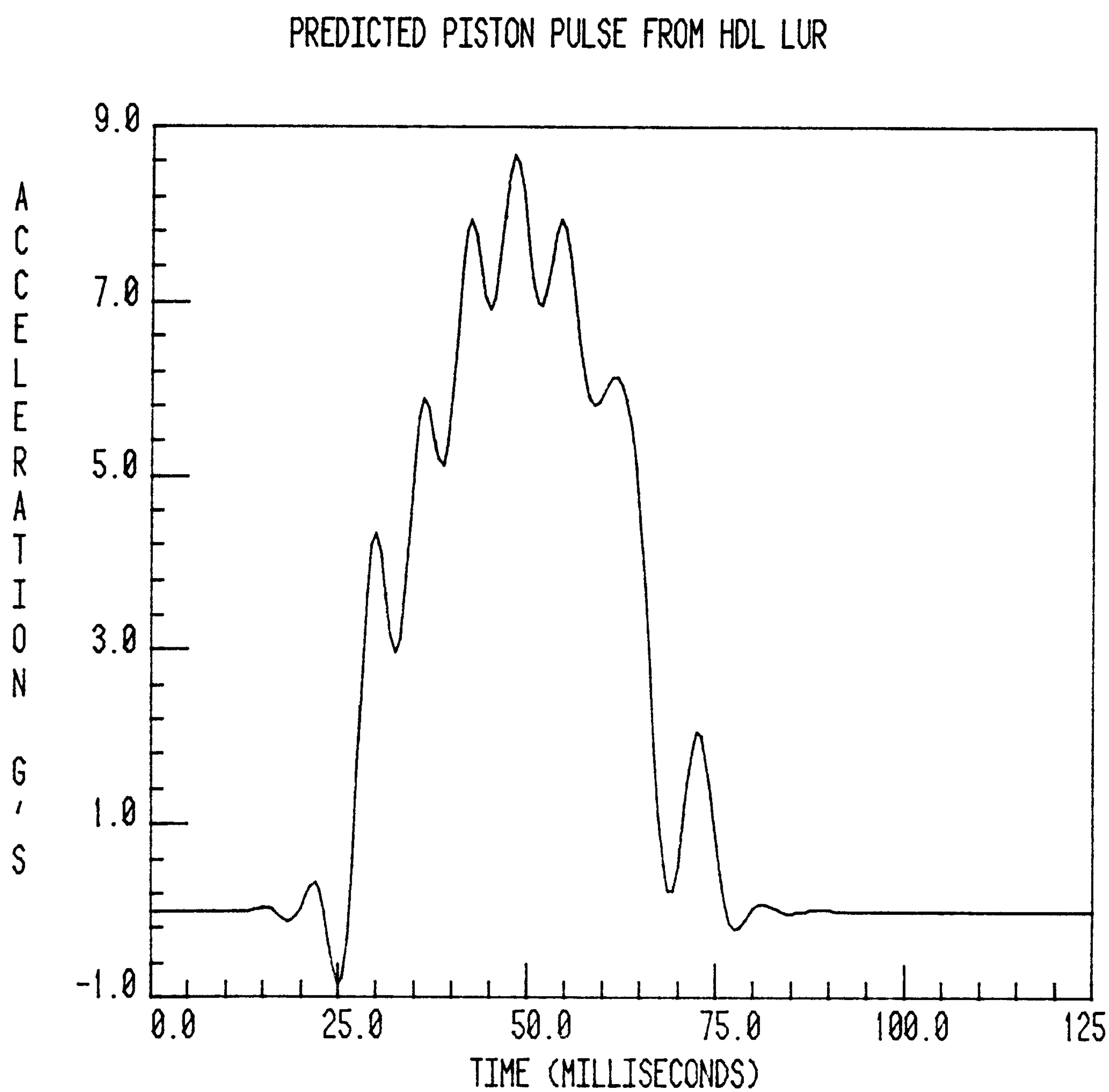
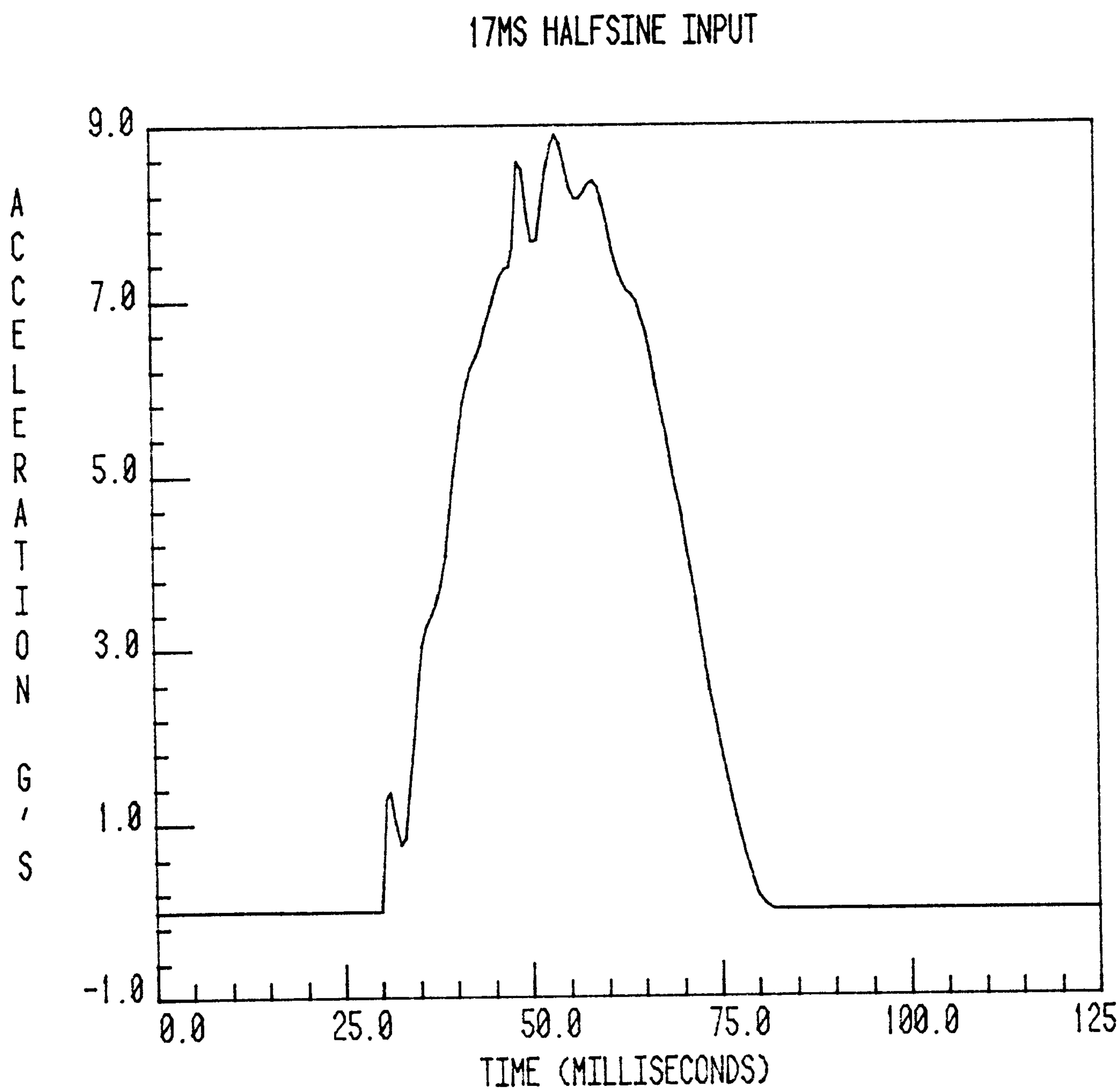

LEFT UPPER RIB FROM HEIDELBERG
ACCELERATION G'S
200.0
100.0
0.0
-100.0
-200.0
-300.0
0.0
25.0
50.0
75.0
100.0
125
TIME (MILLISECONDS)

Figure 5



PREDICTED PISTON PULSE FROM HDL LUR
ACCELERATION G'S
9.0
7.0
5.0
3.0
1.0
-1.0
0.0
25.0
50.0
75.0
100.0
125
TIME (MILLISECONDS)

Figure 6


17MS HALFSINE INPUT
9.0
7.0
5.0
3.0
1.0
-1.0
ACCELERATION G'S
0.0
25.0
50.0
75.0
100.0
125
TIME (MILLISECONDS)